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# ROUGHNESS FUNCTIONS, THE THERMOHYDRAULIC PERFORMANCE OF ROUGH SURFACES AND THE HALL TRANSFORMATION—AN OVERVIEW

M. J. LEWIS
Engineer, EIR Würenlingen, CH.

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**Abstract**—A critical review is presented of the methods used to evaluate the thermohydraulic performance of rough surfaces. The fundamental importance of the roughness functions $R(h^+)$ and $g(h^+, Pr)$ is pointed out. Optimisation is discussed. The Hall transformation, which is used to relate measurements in annuli to circular tube or rod-bundle flows, is re-examined and the implications of some approximations explained. A consistent approach for evaluating the performance of rough surfaces is suggested.

## NOMENCLATURE

$A$, constant in log-law ($\approx 2{\cdot}5$);
$b$, characteristic width of roughness element;
$D$, characteristic hydraulic diameter;
$g(h^+, Pr)$, heat-transfer roughness function defined as a wall boundary condition for a temperature profile;
$h$, characteristic height of roughness element;
$h^+$, roughness Reynolds number $hu_\tau/v$;
$M$, Mach number;
$p$, characteristic pitch of roughness elements;
$Pr$, Prandtl number;
$R(h^+)$, momentum-transfer roughness function defined as a wall boundary condition for a velocity profile;
$St$, Stanton number;
$Stx$, Stanton number multiplier ($St$ divided by smooth wall $St$ at same Reynolds number);
$t^+$, dimensionless fluid temperature;
$u^+$, dimensionless fluid velocity $u/u_\tau$;
$u_\tau$, shear velocity $\sqrt{(\tau_w/\rho)}$;
$y^+$, dimensionless distance from surface $yu_\tau/v$.

Greek symbols

$\lambda$, friction factor;
$\lambda x$, friction factor multiplier ($\lambda$ divided by smooth wall $\lambda$; at same Reynolds number);
$v$, viscosity;
$\rho$, density;
$\tau_w$, wall shear stress.

## 1. INTRODUCTION

THE PURPOSE of this report is to summarize, and hopefully to clarify, the present situation with regard to evaluating the thermohydraulic performance of rough surfaces. A need for a re-appraisal becomes clear when it is recognised how wide a range of different correlations is given in the literature [1]. These often give different performances even for similar roughnesses! In order to review the situation, we first point out how the performance of a rough surface is prescribed analytically in terms of the well-known roughness functions $R(h^+)$ and $g(h^+, Pr)$. These functions are fundamental in that they provide local boundary conditions for the flow field. Methods for measuring the roughness functions are discussed. Because most experiments on rough surfaces have been conducted using heated annuli, a critical look at the Hall transformation [2] is made. It is stressed that when this transformation is employed a temperature profile must be measured. Some commonly used simplifications of the transformation are discussed and the implications of some approximations pointed out. Finally, a consistent approach to the problem of evaluating the performance of rough surfaces is suggested.

## 2. THE ROUGHNESS FUNCTIONS $R(h^+)$ AND $g(h^+, Pr)$ AND THEIR SIGNIFICANCE

Nikuradse [3] defined the momentum-transfer roughness function $R(h^+)$ and Dipprey and Sabersky [4] defined the heat-transfer roughness function $g(h^+, Pr)$. We retain the rather vague nomenclature [see equations (1) and (2)] for the roughness functions because this is how they are generally identified in the literature. These functions are the parameters which describe the thermohydraulic performance of a rough surface. To understand this we consider the fully-developed turbulent flow of a moderate Prandtl number ($Pr$) homogeneous fluid through a channel. It is a well-known practice [5] to divide the flow into two

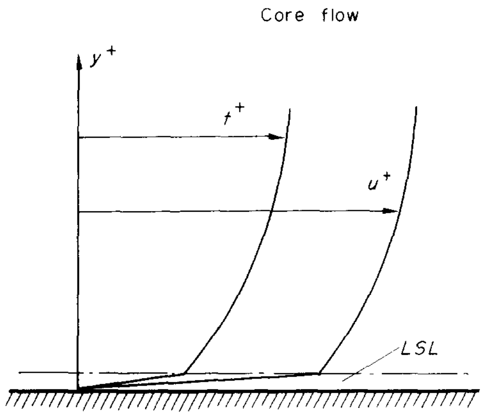


FIG. 1. A smooth wall and the laminar sub-layer (L.S.L.).

or more regions. Typically, for a smooth wall, these may be a laminar or viscous sub-layer and a turbulent outer region or core; Fig. 1. Each region is analysed separately and they are "matched" at their boundaries by having the same velocity or temperature at these boundaries. This division is a von Karman integral approach in the sense that velocity and temperature profiles in the regions are only approximate. Where the matching occurs the most important boundary conditions are satisfied, namely, equal velocity and temperatures but not necessarily equal gradients.

In the case of a hydraulically rough surface, Fig. 2, a laminar sub-layer no longer exists, at least, not at every point on the surface. The momentum- and heat-transfer characteristics of the surface are then controlled by the local separation and re-attachment of the flow at the roughness elements. To avoid considering these mechanisms roughness functions are defined. In essence, $R(h^+)$ is a dimensionless fluid velocity at the edge of a control volume enclosing the roughness elements. It accounts for the momentum losses caused by the roughness and provides a velocity boundary condition for the core flow velocity $u^+$. The function $g(h^+, Pr)$ is essentially a dimensionless temperature difference over the same control volume. Hence, it defines the heat-transfer capabilities of the rough surface and provides a temperature boundary condition for the core flow temperature $t^+$. Both $R$ and $g$ are convenient mathematical functions having little physical significance. They replace the normally accepted wall boundary conditions, $u^+ = t^+ = 0$. $g(h^+, Pr)$ may also be considered as the inverse of either a local average heat-transfer coefficient or a wall Stanton number. Lewis [6, 7] reviews the definition of the roughness functions and summarizes their limitations.

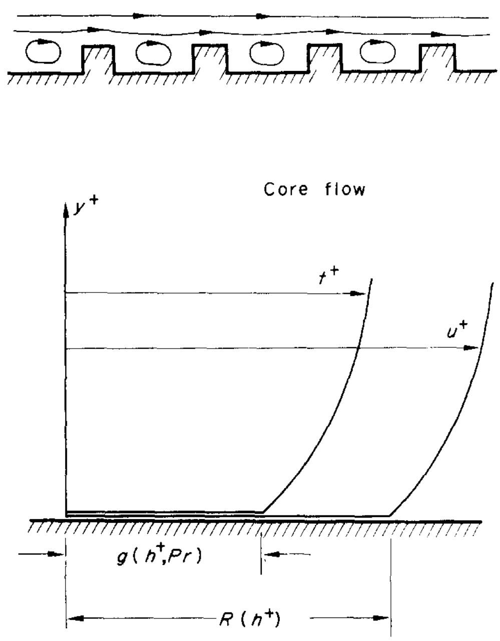


FIG. 2. A rough wall and its analytic representation in terms of roughness functions acting as boundary conditions to the core flow profiles.

Strictly, for any rough surface, we should write

$$R = R\left(h^+, \frac{h}{b}, \frac{h}{p}, \frac{h}{D}, \text{shape}, T, M\right) \tag{1}$$

and

$$g = g\left(h^+, \frac{h}{b}, \frac{h}{p}, \frac{h}{D}, \text{shape}, Pr, T, M\right) \tag{2}$$

where $h^+$ is the well-known roughness Reynolds number, $h$, $p$ and $b$ are lengths which characterise the dimensions and distribution of the roughness elements, $D$ is a hydraulic diameter and $T$ and $M$ are a characteristic temperature and Mach number, respectively. We consider an incompressible, constant property fluid. If $h/D$ is small and if the fluid's Prandtl number is of order unity or greater, the following experimentally verified and widely accepted [5] hypothesis may be postulated:

$$R = R\left(h^+, \frac{h}{b}, \frac{h}{p}, \text{shape}\right) \tag{3}$$

and

$$g = g\left(h^+, \frac{h}{b}, \frac{h}{p}, \text{shape}, Pr\right). \tag{4}$$

Equations (3) and (4) imply that the roughness functions are invariant with a change in the core flow geometry. This is a far-reaching simplification because it means that the surface characteristics can be separated from the core or outer flow region. If $h/D$ is not small the simplification cannot be made. Then both $R$ and $g$

lose their local character and some other analysis, which takes into account the acceleration of the flow for example, must be considered.

With such a simplification the roughness functions may be determined in any experimental rig or wind tunnel. Since they are assumed invariant under a change in the core flow geometry they will provide the foundations, as boundary conditions, for the analytic solutions of the core or outer flow in any system. That is, once $g(h^+, Pr)$ and $R(h^+)$ are established for surfaces of interest, Stanton numbers ($St$), friction factors ($\lambda$) etc. may be determined for any geometry—circular tube, annulus, rod-bundle—provided that eddy diffusivity information is available for the core or outer flow. The overall solution to a problem would then fall into the category of a mean-velocity-field-closure turbulence model for the core, matched to the boundary-value roughness functions.

### 3. MEASUREMENT OF $R(h^+)$ AND $g(h^+, Pr)$

The common approach is to choose a roughness shape and to determine the roughness functions by measurement for a range of $h/b$, $h/p$ and $h^+$. We may separate the measurements into two types; integral or "bulk" methods [5] and local methods [8] which involve measuring velocity and temperature profiles. Bulk methods introduce uncertainties because some model for the core flow must be assumed. By subtracting the core flow solutions from the bulk measurements the roughness functions are determined. Local measurements, which provide values for $R(h^+)$ and $g(h^+, Pr)$ directly, are difficult to perform accurately in small test sections and introduce "origin" uncertainties [9].

For bulk measurements in circular tubes, the analysis is trivial and we obtain [6, 7]

$$R(h^+) = \sqrt{(8/\lambda)} + A \ln 2h/D + 3A/2 \tag{5}$$

and

$$g(h^+, Pr) = R(h^+) + (\lambda/8St - 1)/\sqrt{(\lambda/8)} \tag{6}$$

where $A$ is constant ($\approx 2{\cdot}5$), $\lambda$ and $St$ are the bulk measured quantities and, for example, the term $A \ln 2h/D + 3A/2$ is the velocity-profile core flow "solution". Various uncertainties have been omitted from equations (5) and (6), but these may be shown to be small for a circular tube. Because of the problems involved with manufacturing rough surfaces inside circular tubes, annuli, with the outer surfaces of the inner tubes roughened, have found widespread use. However, the flow in an annulus is a flow in a multiply connected region. That is, it has two independent boundary conditions for velocity and temperature at the inner and outer walls. If the outer surface is adiabatic and the inner wall heated, asymmetry is introduced into the problem and the velocity and temperature profiles are no longer similar. Quarmby and Anand [10] describe a method for modelling the core flow in an asymmetric situation. They solve the momentum and energy equations using an eddy diffusivity model, they satisfy the relevant boundary conditions and incorporate the heat transferred across the zero shear surface. Although their analysis is for an annulus with smooth walls, its extension to rough walls is straightforward. This extension has been employed by Lawn [9]. Other workers (Wilkie [11], for example) use approximate solutions to the core flow and Hall's transformation.

### 4. THE HALL TRANSFORMATION

The Hall transformation [2] is employed to relate measurements in annuli, with roughened inner surfaces, to roughened circular tube or rod-bundle flows. Its usefulness arises from the fact that the flow in the neighbourhood of a rod in an infinite, widely-spaced, symmetric rod-bundle is similar to the flow in a circular tube. But rough surfaces are more easily machined in annuli. Since annuli measurements cannot be used directly, because of asymmetries, transformations are necessary. For an incompressible, constant property fluid, the energy equation is linear and the principle of superposition is valid. This is the basis for the Hall transformation, which circumvents the problem of solving analytically the differential equations for the core flow and temperature field in the annulus. In this transformation the influence of rough walls on the momentum and heat-transfer characteristics of the flow is incorporated directly into the integral quantities ($\lambda$, $St$) and roughness functions are not considered. Hall introduces a transformed inner region of the annulus with the same inner wall boundary conditions and the same velocity profile as the untransformed case but he defines a new temperature profile which gives an adiabatic zero-shear surface. Then, the transformed integral quantities for the inner region will correspond to similar parameters for a circular tube with the same boundary conditions as those on the inner wall of the annulus.

It must be clearly understood that, if the principle of superposition is to be used, one fundamental temperature distribution—the inner region profile of the annulus—must be known. Therefore if the Hall transformation is to be employed, to avoid solving the differential equations, this fundamental profile must always be measured. Approximate solutions to the core flow and temperature fields, based only upon bulk measurements, can lead to errors in the transformation.

## 5. APPROXIMATE SOLUTIONS TO THE CORE FLOW IN ANNULUS MEASUREMENTS

Many empirical models (see Warburton and Pirie [12], for a review) have been proposed to approximate the roughness functions and the annulus core flow solutions needed for the Hall-transformed parameters. Maubach [13] applies an integral method to determine $R(h^+)$. This gives consistent (for the same roughness shape and at the same value of $h^+$, the values of $R(h^+)$ determined by Maubach are the same in an annulus and in a circular tube) results because, by definition, no nett momentum is transferred across the zero-shear surface. However, heat is transferred across this surface which leads to modelling problems for the energy equation.

A problem with the approximations involved in the transformations is to assess their accuracy. This can be done when it is realised that the roughness functions $R(h^+)$ and $g(h^+, Pr)$ re-calculated [15] from the transformed annulus information should be the same as the circular tube functions for similar roughness shapes and at the same values of $h^+$. It was the discrepancies here [16] which led to the present re-appraisal.

It is clear that, because the roughness functions are intimately connected with the velocity and temperature profiles in the flow field, transformations, such as Hall's, which rely upon the energy integral equation, must have (i) the values of $R(h^+)$ and $g(h^+, Pr)$, and (ii) the solution of the core flow velocity and temperature field with the correct boundary conditions, before the transformation can be effected! This essential information could be supplied if all of these quantities were measured but, such measurements would make transformations redundant [9]. Seen in this light, the fundamental arguments for Hall transformed quantities should be re-examined.

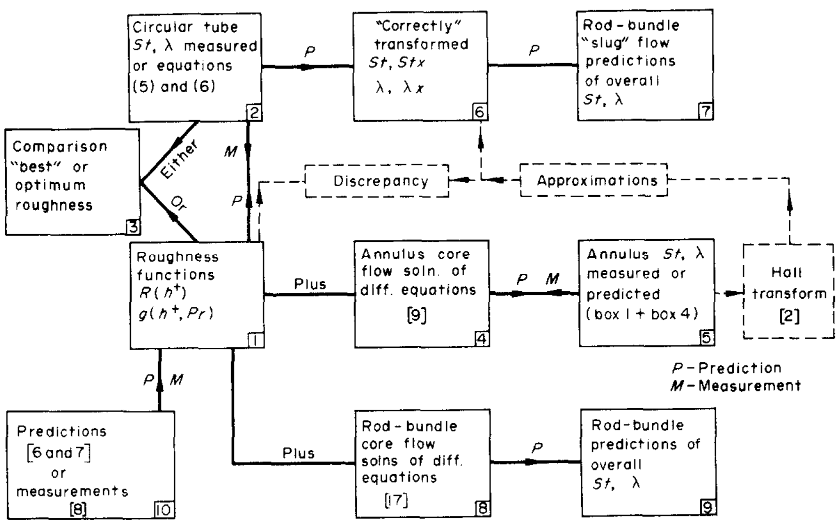


FIG. 3. A consistent approach to evaluating the thermohydraulic performance of a rough surface for an incompressible constant-property fluid.

The empirical expressions used to prescribe the core-temperature boundary conditions [i.e. $g(h^+, Pr)$] and to approximate the energy integrals of the core flow in the transforming equation are not evaluated by an independent measurement. Hence, three empirical quantities—the transformed Stanton number, the boundary conditions and the energy integrals—are being correlated by one measurement of bulk Stanton number. Furthermore, where an eddy diffusivity model is not employed, a modified Reynolds analogy is used to relate the energy integrals of the core flow to the friction factor. But, Reynolds analogy is not valid at or near the zero shear surface and will not account for energy transfer across this surface. An attempt by Kjellström [14] to model the roughness function $g(h^+, Pr)$ using a laminar sublayer concept and Reynolds analogy is conceptually not valid.

## 6. A CONSISTENT APPROACH TO EVALUATING THE PERFORMANCE OF ROUGH SURFACES

Once it is recognised that the roughness functions remain invariant under a change of core flow geometry, a consistent approach to the analysis of such surfaces may be realised. The flow chart of Fig. 3 is presented to summarize this approach.

The first step (box 1) is to determine, experimentally or theoretically, the roughness functions $R(h^+)$ and $g(h^+, Pr)$ for a given roughness shape and distribution. If these functions are to be determined solely by experiment no consideration need be given to the mechanisms operating at the roughness elements. However, in order to optimise or make a proper choice of surface an almost unlimited number of experiments need to be performed, even for a uniformly distributed roughness. An attempt has been made [6, 7] to prescribe the local separated flow over the roughness elements, to analyse this flow and predict the roughness functions (box 10). Changes of shape and the distribution of the elements are considered in the analysis. Such an analysis may be used as a preliminary guide to determine which surfaces are of greatest interest. Experiments may then be employed to qualify the predictions.

The roughness functions may be determined from bulk measurements (box 2) in circular tubes plus equations (5) and (6). On the other hand, local or direct measurements (box 10) may be made. If bulk measurements in annuli are performed (box 5), the eddy diffusivity model (box 4) should be used for the annulus core-flow solution. It is unwise to calculate the function $g(h^+, Pr)$ from transformed integral quantities and equations (5) and (6) because the function will contain errors introduced by the transformation.

Once the roughness functions have been determined they may be directly compared (box 1). Alternatively, equations (5) and (6) may be employed together with the known roughness functions to compare circular tube Stanton numbers and friction factors (box 2) at a fixed $h/D$ and channel Reynolds number. An optimum roughness (box 3) may be evaluated through either of these procedures. The circular tube bulk quantities (box 2), or their multipliers $Stx$ and $\lambda x$, may be considered as "correctly" transformed parameters (box 6) for use in slug-flow rod bundle codes (box 7) for example.

Alternatively, the roughness functions could be used as boundary conditions for rod-bundle [17] core-flow solutions (box 8) to provide overall rod-bundle information (box 9). No extra information is added by employing Hall transformed quantities, shown by the dashed lines.

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## FONCTIONS DE RUGOSITE, PERFORMANCES THERMIQUES ET DYNAMIQUES DES SURFACES RUGUEUSES ET TRANSFORMATION DE HALL—VUE D'ENSEMBLE

**Résumé**—On présente une analyse critique des méthodes d'évolution des performances thermiques et dynamiques des surfaces rugueuses. On met en évidence l'importance fondamentale des fonctions de rugosité $R(h^+)$ et $g(h^+, Pr)$. On discute de l'optimisation. La transformation de Hall, utilisée pour relier les mesures dans l'espace annulaire aux écoulements dans un tube circulaire ou une grappe de barres, est examinée et on explique les implications de quelques approximations. On suggère une approche positive pour évaluer les performances des surfaces rugueuses.

## RAUHIGKEITSFUNKTIONEN, DIE THERMOHYDRAULISCHEN EIGENSCHAFTEN RAUHER OBERFLÄCHEN UND DIE HALL-TRANSFORMATION—EIN ÜBERBLICK

**Zusammenfassung**—In einer kritischen Übersicht werden die Methoden zur Ermittlung des Wärmeübergangs und des Strömungsverhaltens an rauhen Oberflächen gegenübergestellt. Es wird auf die grundlegende Bedeutung der Rauhigkeitsfunktionen $R(h^*)$ und $g(h^*, Pr)$ hingewiesen. Möglichkeiten der optimalen Auslegung werden diskutiert. Die Hall-Transformation, die angewendet wird, um Ergebnisse für den Ringspalt zu Messungen mit Kreisrohren und Stabbündeln in Beziehung zu setzen, wird nachgeprüft und die Bedeutung einiger Annahmen aufgezeigt. Zur Berechnung der thermohydraulischen Vorgänge wird ein Näherungsverfahren vorgeschlagen.

## ПАРАМЕТРЫ ШЕРОХОВАТОСТИ, ТЕПЛОВАЯ И ГИДРАВЛИЧЕСКАЯ ХАРАКТЕРИСТИКИ ШЕРОХОВАТЫХ ПОВЕРХНОСТЕЙ, ПРЕОБРАЗОВАНИЕ ХОЛЛА

**Аннотация** — Приводится критический обзор методов определения тепловой и гидравлической характеристик шероховатых поверхностей. Отмечается фундаментальное значение параметров шероховатости $R(h^+)$ и $g(h^+, Pr)$. Рассматриваются оптимальные характеристики. Анализируется применение преобразования Холла для кольцевого канала к случаю течения в круглой трубе или пучках труб, и даётся обоснование некоторых приближений. Предлагается рациональный подход для оценки характеристики шероховатых поверхностей.